Aggregate Effects of AIDS on Development

Raül Santaeulàlia-Llopis

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Research Question:

How much does the AIDS epidemic delay economic development?
Some Facts about the AIDS Epidemic in the Sub-Sahara

1. AIDS increases the dependents (children and +60) per worker (by as much as 20-25%).

2. AIDS decreases the rate of growth of the population (by 0.08% for every 1% rise of the share of population infected with HIV).

3. AIDS reduces the expectancy of life (by as much as 15-20 years).

4. AIDS reduces the labor earnings of prime age adults (by as much as 25%).
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Raül Santaeulàlia-Llopis (WUSTL)
What is Economic Development?

Output per Capita

Agricultural Share of Output

Raúl Santaeulalia-Llopis (WUSTL)  Aggregate Effects of AIDS on Development  Hewlett/PRB,Dublin Jan2009
How do I Answer the Research Question?

1. I model economic development with a theory that relates a measure of efficiency, $E_t$, capital accumulation, $K_{t+1}$, and population, $\mu_t$,

\[ K_{t+1} = G(E_t, K_t, \mu_t) \]

2. The population age structure follows the process $\mu_{t+1} = \Gamma_t \mu_t$.

3. I model the AIDS epidemic as a change from $\Gamma_t$ to $\Gamma_{A,t}$.

4. I compute the alternative development path,

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$$K_{A,t+1} = G_A(E_t, K_t, \mu_{A,t})$$
1 If the AIDS epidemic had not existed, per capita income would have been 12% larger in 2008.

2 The AIDS epidemic slows down the transition from agricultural to industrial regimes by 105 years.

3 The impact of the AIDS epidemic on the age distribution of the population accounts for 32% of its effects on development.
My Findings

1. If the AIDS epidemic had not existed, per capita income would have been 12% larger in 2008.

2. The AIDS epidemic slows down the transition from agricultural to industrial regimes by 105 years.

3. The impact of the AIDS epidemic on the age distribution of the population accounts for 32% of its effects on development.
1. If the AIDS epidemic had not existed, per capita income would have been 12% larger in 2008.

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Demographics

- Individuals are indexed by age $i \in \{c, y, o\}$ that respectively represent childhood, young adulthood and old age.

- There is a continuum of identical families that consists of three different generations,

$$\mu_t = \left( N^c_t, N^y_t, N^o_t \right)$$

where $N^i_t$ is the number of members of age group $i$, and $N_t = \sum_i N^i_t$. 
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where $N^i_t$ is the number of members of age group $i$, and $N_t = \sum_i N^i_t$. 
Population Projection Matrix Model

\[ \mu_{t+1} = \Gamma_t \mu_t \]

\[ \Gamma_t = \begin{bmatrix}
1 - \pi^c_t & 0 & 0 \\
\pi^c_t & 1 - \pi^y_t & 0 \\
0 & \pi^y_t & 1 - \pi^o_t
\end{bmatrix} \times \begin{bmatrix}
\gamma^c_t & 0 & 0 \\
0 & \gamma^y_t & 0 \\
0 & 0 & \gamma^o_t
\end{bmatrix} \times \begin{bmatrix}
1 & \phi^y_t & \phi^o_t \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \]

Ageing Matrix \quad Mortality Matrix \quad Fertility Matrix
Life Cycle Graph
Demographics (continued)

\[ N_{t+1}^y = \gamma_t^c \pi_t^c N_t^c + \phi_t^y \gamma_t^c \pi_t^c N_t^y + \gamma_t^y (1 - \pi_t^y) N_t^y + \phi_t^o \gamma_t^c \pi_t^c N_t^o \]
Demographics (continued)

1. Age dependency ratios for children, $\frac{N^c_t}{N^y_t}$, and old age, $\frac{N^o_t}{N^y_t}$.

2. Population Growth Rate, $\frac{N_{t+1} - N_t}{N_t}$.

3. Life Expectancy, is computed as

$$LE_t = \sum_j N_t(j, 1)$$

$$N_t = I + \left[ \Pi_t \times \Gamma_t^M \right] + \left[ \Pi_t \times \Gamma_t^M \right]^2 + \left[ \Pi_t \times \Gamma_t^M \right]^3 + \cdots = (I - \left[ \Pi_t \times \Gamma_t^M \right])^{-1}$$
Demographics (continued)

1. Age dependency ratios for children, $\frac{N_t^c}{N_t^Y}$, and old age, $\frac{N_t^o}{N_t^Y}$.

2. Population Growth Rate, $\frac{N_{t+1} - N_t}{N_t}$.

3. Life Expectancy, is computed as

$$LE_t = \sum_j N_t(j, 1)$$

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Specification of the AIDS Shock

I can model the AIDS epidemic as a shock that changes the fertility rates from $\phi^i_t$ to $\phi^A_{i,t}$, and the mortality rates from $1 - \gamma^i_t$ to $1 - \gamma^A_{i,t}$.

That is, AIDS reshapes the law of motion of the population from $\Gamma_t$ to $\Gamma^A_{A,t}$, generating an alternative population path, $\mu^A_{A,t}$.

This way, AIDS subsequently distorts the population age structure, population growth rates and life expectancy.
1. Malthus Sector (Land is a Fixed Factor),

\[ Y_{mt} = A_{mt} K_{mt}^\phi N_{mt}^\mu L^{1-\phi-\mu} \]

2. Solow Sector,

\[ Y_{st} = A_{st} K_{st}^\theta N_{st}^{1-\theta} \]

3. \( A_{mt} = A_{m0} \gamma_m^t, \ A_{st} = A_{s0} \gamma_s^t \) and \( \gamma_s > \gamma_m \).
Technology

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Development is Inevitable

Marginal Cost

- MC_Solow(Y)
- MC_Malthus(Y ; Ω)
- MC_Malthus(Y ; ~Ω)

with, Ω : 1-μ-φ > μ+φ
Allocations across Sectors (I)

Factor prices equate the marginal productivities across sectors,

\[ w_t = (1 - \theta) A_{st} K_{st}^\theta N_{st}^{-\theta} = \alpha A_{mt} K_{mt}^\phi N_{mt}^{\alpha-1} L^{1-\phi-\alpha} \]  

(1)

\[ r_{Kt} = \theta A_{st} K_{st}^{\theta-1} N_{st}^{1-\theta} = \phi A_{mt} K_{mt}^{\phi-1} N_{mt}^{\alpha} L^{1-\phi-\alpha} \]  

(2)

\[ r_{Lt} = (1 - \phi - \alpha) A_{mt} K_{mt}^\phi N_{mt}^{\alpha} L^{-\phi-\alpha} \]  

(3)
Allocations across Sectors (II):

- Given prices and market clearing conditions, the Malthusian share of total labor, \( n_{mt} = \frac{N_{mt}}{N_{mt} + N_{st}} \), is determined by

\[
F(n_{mt}; E_t, K_t, N_t) = 0
\]

where \( E_t = \frac{A_{st}}{A_{mt}} \). Explicitly,

\[
F(n_{mt}; E_t, K_t, N_t) = \chi_1 E_t \ K_t^{\theta - \phi} \ N_t^{1-\mu-\theta} \ n_{mt}^{1-\mu-\phi} - [1 - (1 - \chi_2) n_{mt}]^{\theta - \phi}
\]

- The capital-labor ratio is not a sufficient statistic.
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- The capital-labor ratio is not a sufficient statistic.
Household problem

Young adults solve,

$$\max_{c_t \geq 0, \ a_{t+1} \in A, \ s_K t \in [0, 1]} \sum_{t=0}^{\infty} \beta^t N_t^\gamma \left[ u^\gamma \left( \frac{c_t}{\eta_y, t} \right) + \beta \gamma^\gamma_t \left( \pi^\gamma_t u^o \left( \frac{c_{t+1}}{\eta_o, t+1} \right) + (1 - \pi^\gamma_t) u^\gamma \left( \frac{c_{t+1}}{\eta_y, t+1} \right) \right) \right]$$

over $c_t \geq 0, a_{t+1} \in A$ and $s_K t \in [0, 1]$, subject to the budget constraint,

$$c_t + a_{t+1} = w_t \epsilon^\gamma_t + \left( (1 + r_{Kt} - \delta) s_K t + \frac{q_t + r_{Lt}}{q_{t-1}} (1 - s_K t) \right) a_t$$
Equilibrium

Given the initial capital stock, $K_0$, the amount of land $L_0(= 1)$, the sequence of the AIDS epidemic, the sequence of the distribution of the population, $\{\mu_t\}_{t \geq 0}$, a competitive equilibrium in this economy consists of sequences of prices $\{q_t\}_{t \geq 0}$ and $\{w_t, r_{K,t}, r_{L,t}\}_{t \geq 0}$, sequences of firm allocations, $\{K_{mt}, K_{st}, N_{mt}, N_{st}, Y_{mt}, Y_{st}\}_{t \geq 0}$, sequences of household allocations, $\{c_t, a_{t+1}, s_{Kt}\}_{t \geq 0}$, the sequence of the distribution of the population, $\{\mu_{A,t}\}_{t \geq 0}$ such that the following are true:

1. Firms optimize: $w_t, r_{K,t}$ and $r_{L,t}$ equate marginal productivities across sectors, (1)-(3).

2. Households optimize: $c_t, a_{t+1}, s_{Kt}$ solve the utility maximization problem, (4).

3. Goods market clears, and

   \[ K_{mt} + K_{st} = K_t \]
   \[ N_{mt} + N_{st} = N_t \]

4. Population evolves as $\{\mu_{A,t}\}_{t \geq 0}$. 
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Malthus Sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{m0}$</td>
<td>1.0</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.6</td>
<td>Egypt Labor Share in 1950</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.1</td>
<td>Egypt Land Share in 1950</td>
</tr>
<tr>
<td><strong>Solow Sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{s0}$</td>
<td>0.45</td>
<td>Matches agricultural share of Egypt in 1990</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.29</td>
<td>U.S. Capital Share</td>
</tr>
<tr>
<td>$\gamma^s$</td>
<td>1.85</td>
<td>U.S. TFP growth</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.08</td>
<td>U.S. Depreciation</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>U.S. K/Y</td>
</tr>
</tbody>
</table>

$N_t^c, N_t^y$ and $N_t^o$ are the actual pre-2006 population groups + post-2006 U.N. Population Division projections.

Malthusian TFP growth rate, $\gamma_t^m$, is calibrated such that income per capita growth in the Malthus era is zero.

I use OECD equivalence scales: $\eta_t = 0.5 \ N_t^c + 0.5 \ N_t^y + 0.7 \ N_t^o$. 
This Theory is Useful to Understand Development in Africa

Figure: Income Per Capita, Model and Data.
Alternative Counterfactual?

Lesotho (23.2% HIV+)

Egypt (<.1% HIV+)

Malawi (12.7% HIV+)

Kenya (6.1% HIV+)
Projection of Adult $HIV_t$ Prevalence Rate

Projection of HIV-Prevalence Rate with HIV Rate = 0% at 2050

Lesotho
Malawi
Kenya
AIDS increases the Age-Dependency Ratio

Age-Dependency Ratio with AIDS / without AIDS

- Swaziland (25.3% HIV+)
- Sub-Saharan Africa (5.9% HIV+)

Graph showing the Age-Dependency Ratio with AIDS and without AIDS for Swaziland and Sub-Saharan Africa from 2000 to 2050.
AIDS reduces Labor Efficiency

Efficiency Units of Labor
MDICP-2006, weighted with Rural Age-Sex Structure of DHS-2004

- Single HIV-
- Single HIV+
- Married: Both HIV-
- Married: HIV+ Woman & HIV- Man
- Married: HIV- Woman & HIV+ Man
- Married: Both HIV+

Man (dark green) and Woman (light green) efficiency units.
Results from Infecting Egypt with Lesotho’s AIDS shock in 1981

Egypt was 22% Agricultural in 1981

Raúl Santaeulalia-Llopis (WUSTL)
Results from Infecting Egypt with Lesotho’s AIDS shock in 1972

Egypt was 43% Agricultural in 1972 (= Lesotho in 1981)
Results from Infecting Egypt with Malawi’s AIDS shock in 1964
Egypt was 72% Agricultural in 1964 (≈ Malawi in 1981)
Impact of the Age Structure

AIDS Agricultural Share - Benchmark Agricultural Share

-0.05
0.00
0.05
1959
1969
1979
1989
1999
2009
2019
2029
2039
2049
2059
2069
2079
2089
2099

With AIDS affecting the Age Structure
With AIDS not affecting the Age Structure
Misspecification of the AIDS Shock: The Black Death
The Black Death (Overnight Killing of 50% of the population) in the Solow Phase

- Population shocked by the Black Death
- Population Benchmark

- Income per capita (Black Death), Index = 1 at time=0
- Income per capita (Benchmark), Index = 1 at time=0

- Capital (Black Death)
- Capital (Benchmark)

- Agricultural Share (Black Death)
- Agricultural Share (Benchmark)
The Black Death at Early Stages of Development

- Population shocked by the Black Death
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- Agricultural Share (Black Death)
- Agricultural Share (Benchmark)
Industrialization Delay due to the Black Death
The Black Death Agricultural Share minus the Benchmark Agricultural Share

- Black Death arrives when the Economy is less than 1% Agricultural
- Black Death arrives when the Economy is 30% Agricultural
- Black Death arrives when the Economy is 85% Agricultural

Periods after the Black Death Shock

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Conclusion

1. The effects AIDS on development are much worse than what we think.
   - AIDS reduces income per capita by 12% when HIV is at its highest.
   - AIDS delays economic development for almost a century for countries that are at the earliest stages of development.

2. The impact of AIDS on the age structure generates around 32% of the effects of AIDS on Development.

3. If we do not succeed in promoting better health in poor countries, and in Sub-Saharan Africa this means primarily getting rid of AIDS, we should not expect much success from other policies that try to fix the problem of persistent poverty.
Further Ongoing Research: toward Policy Evaluation

   
   ”AIDS and Development: The Quantitative Role of RH Policies”

2. Endogenous Fertility, Life Expectancy and Human Capital:
   
   ”AIDS, Population and Human Capital”, with Rody Manuelli.

3. Endogenous Sexual Behaviour:
   
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AIDS and Economic Development:
The Quantitative Role of Reproductive Health Policies

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3rd Annual Research Conference on Population,
Reproductive Health and Economic Development,
Hewlett/PRB, Dublin Jan 2009
MOTIVATION: Current Allocation of Global Funds to Fight AIDS

- The largest donor to fight AIDS is the United States (about 40% of all global funds committed for HIV/AIDS in 2005)

- U.S. President’s Emergency Plan for AIDS Relief (PEPFAR)\(^1\)
  - 55% for the treatment of individuals with HIV/AIDS – of which 75% between financial years 2006-2008 must be spent on the purchase and distribution of antiretrovirals (ARTs).
  - 15% for the palliative care of individuals with HIV/AIDS.
  - 10% for helping orphans and vulnerable children.
  - 20% for HIV/AIDS prevention – of which 33% must be directed at abstinence until marriage programmes.

- The Bill and Melinda Gates Foundation – the largest philanthropic funder in the HIV/AIDS arena – follows similar guidelines – Global HIV Vaccine Enterprise.

\(^1\)An umbrella organization for all existing US AIDS programs. PEPFAR is a 5-year, $15 billion, global initiative to combat AIDS and it represents a growing share of overall U.S. foreign assistance.
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¹An umbrella organization for all existing US AIDS programs. PEPFAR is a 5-year, $15 billion, global initiative to combat AIDS and it represents a growing share of overall U.S. foreign assistance.
The current allocation of funds to fight AIDS:

1. places the core strategy of AIDS-related policies on the treatment of *AIDS-progression* – mostly via the distribution of ARTs (*70% of all funds*), and

2. leaves very little role for Reproductive Health and Family Planning (RH/FP) prevention policies focused on reductions of *HIV-infection* (*20% of all funds*).
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However...

- AIDS is more than reductions in life expectancy (the margin ARTs target).

- We have seen AIDS also represents:
  - changes the age-structure of populations,
    (more than 80% of HIV infections are due to heterosexual intercourse)
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2. changes fertility and the population growth rate,
In addition, importantly, AIDS also represents an unprecedented seismic shift to the structure of families:

1. Because AIDS kills mostly young adults, it also kills spouses and parents generating large numbers of widows and orphans.

2. 2 out of 3 of the HIV-infected couples are discordant couples and between 30%-40% of the infected couples are couples where the female partner only is infected.

3. AIDS is associated with higher divorce rates and marital reshuffling.

4. AIDS is associated with delays in first marriages.

These margins can push further the strain of poor households (widows, divorced and singles), lowering saving rates.
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To Sum Up

- There are potentially strong economic arguments that point to integrated RH/FP and AIDS policies targeting prevention of \textit{HIV-infection}.\footnote{The range of integrated RH/FP and AIDS policies targeting prevention of \textit{HIV-infection} is wide: reduce \textit{HIV-infection} of adults by counselling on the use of condoms, circumcisions, treatment of genital soars or abstention; increase prevention of mother-to-child \textit{HIV-infection} at pregnancy, delivery and breastfeeding; and, identify, support and counsel the planning of families that foster or care for AIDS orphans.}

- However, we do not know yet the effects of each of the AIDS-related polices on the development path, neither those RH/FP policies that focus on prevention to reduce \textit{HIV-infection} nor those that currently focus on ARTs treatment to reduce AIDS-progression. This is the task of this ongoing research.
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How do I plan to Answer the Question?

- Build a population model - a **demography of HIV and AIDS** - on the basis of a large but finite number of individuals and family types that can be used to:

  1. *identify* the effects that AIDS has on individuals and their family’s composition over time isolating those due to *HIV-infections* from those due to *AIDS-progression*, and

  2. *consistently aggregate* the demographics of a society (with AIDS and no-AIDS scenarios) in a tractable manner that we can incorporate into a development theory to provide aggregate economic outcomes - income per capita and agricultural share of output.

The fact that I can separately identify *HIV-infection* rates from *AIDS-progression* rates on individuals allows me to evaluate – in terms of economic outcomes – the effect of each type of policy in isolation.
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The fact that I can separately identify HIV-infection rates from AIDS-progression rates on individuals allows me to evaluate – in terms of economic outcomes – the effect of each type of policy in isolation.
To capture all relevant types of households and its dynamics, I define a household as a vector $z_t$ that subsumes the age, sex, marital status and health status of each of its individuals in period $t$. 
Definition of a Family

A family consists of a set of people of up to three different generations with at most an elderly couple, and adult couple and maximum number \( N \) of children.

Formally, a household is a vector that states the relevant properties to each member current demographic fate,

\[
    z = \left( z^{1}, z^{2f}, z^{2m}, z^{3f}, z^{3m} \right) \in Z = [0, N] \times \{0, 1, 2, 3\}^2 \times \{0, 3\}^2
\]  

The individual state of young adults may take different values depending on his/her health status, \( z^{2g} \in \{0, 1, 2, 3\} \), respectively deceased, AIDS-progressed, HIV-infected (HIV+), or healthy (HIV-).

Any proper subset is also a household type. Note the information on the health status of each member is incorporated in the household type.
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$$
\mathbf{z} = \left( z^1, z^2_f, z^2_m, z^3_f, z^3_m \right) \in \mathbb{Z} = [0, N] \times \{0, 1, 2, 3\}^2 \times \{0, 3\}^2 \tag{4}
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Any proper subset is also a household type. Note the information on the health status of each member is incorporated in the household type.
Specification of the Population Process

To specify the population process I formulate a population matrix projection model,

$$\mu^{t+1} = \Gamma \mu^t$$  \hspace{1cm} (5)

Hence, the measure of families of a particular type $z'$ at period $t + 1$ is given by

$$\mu_{z'}^{t+1} = \sum_z \Gamma_{z'|z} \mu_z^t$$
Population Process of Young Adults without Dependents

\[
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_2 \\
\mu_3
\end{bmatrix}^{t+1} = \begin{bmatrix}
1 & 1 - \gamma_2^{AIDS} & 1 - \gamma_2^{HIV} & 1 - \gamma_2 \\
0 & \gamma_2^{AIDS} & \lambda_2^{HIV} & 0 \\
0 & 0 & (1 - \lambda_2^{AIDS})\gamma_2 & 0 \\
0 & 0 & 0 & (1 - \lambda_2^{HIV})\gamma_2
\end{bmatrix} \begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_2 \\
\mu_3
\end{bmatrix}^{t}
\]

The most important bit here is that we can separately identify the HIV-infection rates, \( \lambda^{HIV} \), from the AIDS-progression rates, \( \lambda^{AIDS} \).

Where \( \gamma_2 \) is the mortality rate of a young adult individual of sex \( g \) that is not HIV-infected, \( \gamma_2^{HIV} \) is the mortality rate of that individual when it is HIV-infected, and \( \gamma_2^{AIDS} \) is the mortality rate of an individual that has developed AIDS.
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\[
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_2 \\
\mu_3
\end{bmatrix}^{t+1} =
\begin{bmatrix}
1 & 1 - \gamma_{2g}^{AIDS} & 1 - \gamma_{2g}^{HIV} & 1 - \gamma_{2g} \\
0 & \gamma_{2g}^{AIDS} & \lambda_{2g}^{AIDS} & 0 \\
0 & 0 & (1 - \lambda_{2g}^{AIDS}) \gamma_{2g} & \gamma_{2g}^{HIV} \\
0 & 0 & 0 & (1 - \lambda_{2g}^{HIV}) \gamma_{2g}
\end{bmatrix}_{4 \times 4}
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_2 \\
\mu_3
\end{bmatrix}^t
\]

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0 & 0 & (1 - \lambda_{2g}^{AIDS}) \gamma_{2g} & 0 \\
0 & 0 & 0 & (1 - \lambda_{2g}^{HIV}) \gamma_{2g} \\
\end{bmatrix}_{4 \times 4}
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_2 \\
\mu_3 \\
\end{bmatrix}^t
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The most important bit here is that we can separately identify the **HIV-infection** rates, \(\lambda^{HIV}\), from the **AIDS-progression** rates, \(\lambda^{AIDS}\).

Where \(\gamma_{2g}\) is the mortality rate of a young adult individual of sex \(g\) that is not HIV-infected, \(\gamma_{2g}^{HIV}\) is the mortality rate of that individual when it is HIV-infected, and \(\gamma_{2g}^{AIDS}\) is the mortality rate of an individual that has developed AIDS.
Survival Matrix of Young Adults

\[
\begin{bmatrix}
1 & 1 - \gamma_{2f}^{AIDS} & 1 - \gamma_{2f}^{HIV} & 1 - \gamma_{2f} \\
0 & \gamma_{2f}^{AIDS} & \lambda_{2f}^{AIDS} & 0 \\
0 & 0 & (1 - \lambda_{2f}^{AIDS})\gamma_{2f}^{HIV} & 0 \\
0 & 0 & 0 & (1 - \lambda_{2f}^{HIV})\gamma_{2f}^{HIV}
\end{bmatrix}
\otimes
\begin{bmatrix}
1 & 1 - \gamma_{2m}^{AIDS} & 1 - \gamma_{2m}^{AIDS} & 1 \\
0 & \gamma_{2m}^{AIDS} & \lambda_{2m}^{AIDS} & 0 \\
0 & 0 & 0 & (1 - \lambda_{2m}^{AIDS})\gamma_{2m}^{AIDS} \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[4 \times 4\]
Surival Matrix of Old Adults

\[
\begin{bmatrix}
\mu_{z,0,0} \\
\mu_{z,0,3} \\
\mu_{z,3,0} \\
\mu_{z,3,3}
\end{bmatrix}^{t+1} = \\
\begin{bmatrix}
1 & 1 - \gamma_{3f} \\
0 & \gamma_{3f}
\end{bmatrix}_{2\times2} \otimes \\
\begin{bmatrix}
1 & 1 - \gamma_{3m} \\
0 & \gamma_{3m}
\end{bmatrix}_{2\times2}^{4\times4} \\
\begin{bmatrix}
\mu_{z,0,0} \\
\mu_{z,0,3} \\
\mu_{z,3,0} \\
\mu_{z,3,3}
\end{bmatrix}^t
\]
Survival Matrix of Children

The measure of households with a fixed composition of adult members and children \( z^{1'} \) in \( t + 1 \), is derived as

\[
\mu^t_{z^{1'}, z} = \sum_{z^1} \left[ \Gamma_{z^{1'} | z^1}^{S1} | \bar{Z}^{if}, \bar{Z}^{im} \right] \mu^t_{z^1, z}
\]

where the survival matrix of children, \( \Gamma^{S1} | \bar{Z}^{if}, \bar{Z}^{im} \), conditional on the age, health and marital status of the biological mother is explicitly,
Survival of Children Matrix (continued)

\[
\Gamma^{S1} | \bar{Z}^{if}, \bar{Z}^{im} =
\begin{bmatrix}
1 & 1 - \gamma_{1c} & (1 - \gamma_{2c})^2 & (1 - \gamma_{3c})^3 & \cdots & (1 - \gamma_{Nc})^N \\
0 & \gamma_{1c} & 2\gamma_{2c}(1 - \gamma_{2c}) & 3\gamma_{3c}(1 - \gamma_{3c})^2 & \cdots & N\gamma_{Nc}(1 - \gamma_{Nc})^{N-1} \\
0 & 0 & \gamma_{2c} & 3\gamma_{3c}^2(1 - \gamma_{3c}) & \cdots & N\gamma_{Nc}^2(1 - \gamma_{Nc})^{N-2} \\
0 & 0 & 0 & \gamma_{3c}^3 & \cdots & N\gamma_{Nc}^3(1 - \gamma_{Nc})^{N-3} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \gamma_{Nc}^N \\
\end{bmatrix}
\]
Fertility Matrix

The fertility matrix, $\Phi \mid \bar{z}^{if}, \bar{z}^{im}$, conditional on the age, health and marital status of the biological mother is explicitly,

$$
\Phi \mid \bar{z}^{if}, \bar{z}^{im} = \begin{bmatrix}
1 - \varphi_1 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\varphi_1 & 1 - \varphi_2 & 0 & 0 & \cdots & 0 & 0 \\
0 & \varphi_2 & 1 - \varphi_3 & 0 & \cdots & 0 & 0 \\
0 & 0 & \varphi_3 & 1 - \varphi_4 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1 - \varphi_N & 0 \\
0 & 0 & 0 & 0 & \cdots & \varphi_N & 1
\end{bmatrix}_{(N+1) \times (N+1)}
$$

where $\varphi_j$ is the fertility rate of children when the stock of children in the household is $j$. 
Marriage Process

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \mu_0,0 \\
0 & 0 & 1 & 0 & \mu_0,1 \\
0 & 0 & 0 & 1 & \mu_0,2 \\
0 & 0 & 0 & 0 & \mu_1,0 \\
0 & 0 & 0 & 0 & \mu_1,1 \\
0 & 0 & 0 & 0 & \mu_1,2 \\
0 & 0 & 0 & 0 & \mu_2,0 \\
0 & 0 & 0 & 0 & \mu_2,1 \\
0 & 0 & 0 & 0 & \mu_2,2 \\
\end{bmatrix}
\]
Aging Process

I specify a process of aging in which age intervals do not coincide with time (projection) intervals. This falls in the category of population models usually denoted as size-classified life-cycle models.
Aging Process (continued)

The matrix of aging is a \(36(N + 1)\)-by-\(36(N + 1)\) matrix with \(1 - \pi\) in its diagonal. For a given family composition

\[ z_t = (z^1_t, z^{2f}_t, z^{2m}_t, z^{3f}_t, z^{3m}_t) \in Z = [0, N] \times \{0, 1, 2\}^2 \times \{0, 2\}^2, \]

if the family ages, then \(z_t\) disappears and aging creates \(z^1\) new households in \(t + 1\). But not all households age, \(1 - \pi\) will remain in the same stage while \(\pi\) households will age. In detail, if \(z^1 \neq 0\) then a total number of \(z^1\pi\) households will be created as the sum of: \(1\pi\) households of type \(z_{t+1} = (0, z^{2f}_{t+1}, 0, z^{3f}_{t+1}, z^{3m}_{t+1})\) where \((z^{2f}_t, z^{2m}_t) = (z^{3f}_{t+1}, z^{3m}_{t+1})\) (this way, I allocate old adults to one of their daughters); \(\frac{z^1_t - 1}{2}\pi\) households of type \(z_{t+1} = (0, z^{2f}_{t+1}, 0, 0, 0)\); and, \(\frac{z^1_t - 1}{2}\pi\) households of type \(z_{t+1} = (0, 0, z^{2m}_{t+1}, 0, 0)\). If \(z^1 = 0\) and \((z^{2f}_t, z^{2m}_t) \neq (0, 0)\), then \(1 - \pi\) will remain in the same stage, while \(\pi\) households of type \(z_{t+1} = (0, 0, 0, z^{2f}_t, z^{2m}_t)\) will be created. Also, orphans become single HIV-free households with equal gender probability if they age. The rest of elements in this matrix are zeros.
The Population Projection Matrix

We now have all the ingredients to build the population matrix for the whole economy. The population projection matrix is the following $36(N+1)$-by-$36(N+1)$ square matrix

$$
\Gamma_{36(N+1) \times 36(N+1)} = \\
\left[ \Gamma^S_{9 \times 9} \bigotimes \Gamma^S_{4 \times 4} \bigotimes \left[ \Gamma^S_{1 \times 1} \times \Phi \right]_{(N+1) \times (N+1)} \right]_{36(N+1) \times 36(N+1)} \\
\times \Gamma^M_{36(N+1) \times 36(N+1)} \times \Gamma^A_{36(N+1) \times 36(N+1)} \\
\right]
$$

(6)

"The Population Projection Matrix"
Household Problem

Households consist of families with at most three overlapping generations. Young adults of a $z$-type family at period $t$ solve

$$V^y_t(z, a) = \max_{c \geq 0, x^t \in A, s_k \in [0, 1]} u^y_z(c) + \beta \sum_{z' \in Z} \Gamma(z'|z) \{\pi V^o_{t+1}(z', a') + (1-\pi) V^y_{t+1}(z', a')\}$$

subject to the budget constraint,

$$c + x^t = w_t \epsilon_z + \left(\left(1 + r^K_t - \delta\right)s_k + \frac{q_t + r^L_t}{q_{t-1}}(1 - s_k)\right) a$$

The current felicity of young adults is the sum of their utility functions:

$$u^y_z(c) = \sum_{g \neq g^*, g \in G} 1_{z^y, g \neq 0} \omega^y_{i, g} u^y_{z^g} \left(\frac{c}{(1 + n^\nu)\left(\alpha(1_{z^o, m} + 1_{z^o, f})\right)}\right)$$
Old adults are not involved in the savings decision of the household, and their optimal choice is trivial: consume all they get from young adults. Young adults internalize the problem of old adults,

\[
V^o_t (z, a) = \max_{\tilde{c} \in [0, \alpha(1_{z^o, m} + 1_{z^o, f}) \cdot c]} u^o_z (\tilde{c}) + \beta \sum_{z' \in Z} \Gamma (z' | z) (1 - \pi) V^o_{t+1} (z', a')
\]

\[
u^o_z (\tilde{c}) = \sum_{g \neq g^*, g \in G} 1_{z^o, g \neq 0} \omega^{o, i, g} u^o_{z^o, g} \left( \frac{\tilde{c}}{1 + 1_{z^o, g^* \neq 0}} \right)
\]
The Euler Equation

The Euler equation for young agents,

$$\frac{\partial u_z^y(c)}{\partial c} = \beta \sum_{z' \in Z} \Gamma (z'|z) \left\{ (1 + r_{t+1}^K - \delta) \frac{\partial a'(x', x^*)}{\partial x'} \left[ \pi \frac{\partial u^o_{z'}(\tilde{c}')}{\partial c'} + (1 - \pi) \frac{\partial u^y_{z'}(c')}{\partial c'} \right] \right\}$$

with

$$c = w_t \epsilon_z + (1 + r_t^K - \delta) a - x'$$
$$c' = w_{t+1} \epsilon_{z'} + (1 + r_{t+1}^K - \delta) a'(x', x^{*'}) - x''$$
$$\tilde{c} = \alpha (1_{z^o,m} + 1_{z^o,f}) c$$

Since we know function $x''$ at $T$, we can solve for $x'$ backwards. That is, (7) is a functional equation in $x'$. For each period $t$ we have to find a solution for the function $x'(z, a)$. a functional equation in $x_{t+1}(z, a)$, evaluated at every family state $z$ and wealth state $a$. 
Equilibrium

Given the initial distribution of wealth $\chi_0$, the amount of land $L_0 (= 1)$, the initial population structure $\mu_0$ and the population dynamics $\{\Gamma_t\}_{t \geq 0}$, a competitive equilibrium in this economy consists of sequences of prices, $\{q_t\}_{t > 0}$ and $\{w_t, r^K_t, r^L_t\}_{t \geq 0}$, sequences of firm allocations, $\{K_{mt}, K_{st}, N_{mt}, N_{st}, Y_{mt}, Y_{st}\}_{t \geq 0}$, sequences of household functions, $\{c_t, x_{t+1}, s^K_t, V_t\}_{t \geq 0}$, and sequences of wealth distributions $\{\chi_t\}_{t > 0}$ and population measures $\{\mu_t\}_{t > 0}$ such that the following are true:

- Firms optimize: Factor prices $w_t$, $r^K_t$ and $r^L_t$ equate marginal productivities.
- Households optimize: Decision rules $\{c_t, x_{t+1}, s^K_t\}$ generate the value function $\{V_t\}$.
Equilibrium (continued)

- Markets clear:
  \[
  \int_{Z \times A} c_t(z, a) \mu_{z,t} \, d\chi_t + K_{t+1} = Y_t + (1 - \delta)K_t 
  \]  
  (8)

  \[
  A_{t+1} = \int_{Z \times A} \chi_{t+1}(z, a) \mu_{z,t} \, d\chi_t 
  \]  
  (9)

  \[
  K_{t+1} = A_{t+1} - q_t 
  \]

  \[
  N_t = \sum_{z_t} \epsilon_{z_t} \mu_{z_t} 
  \]  
  (11)

  \[
  K_t = K_{mt} + K_{st}, \quad N_t = N_{mt} + N_{st} 
  \]  
  (12)

  \[
  Y_t = A_{mt}F(K_{mt}, N_{mt}, L_t) + A_{st}F(K_{st}, N_{st}) 
  \]  
  (13)

- The wealth distribution that agents use to forecast prices is consistent with individual behavior,

  \[
  \chi_{t+1}(B) = M_t(\chi_t) = \sum_{z} \Gamma(z'|z) \int_B 1_{\chi_{t+1}(z, a) \in B} \chi_t(d(a)) 
  \]  
  (14)

  where, \( M_t \) is the wealth distribution transition law.

- The measure of families of type \( z_t \) evolves according to

  \[
  \mu_{t+1} = \sum_z \Gamma(z'|z) \mu_t 
  \]
Computation Algorithm

I assume the transition from Malthus to Solow takes place after $T$ periods. To compute the equilibrium transition we follow these steps:

- Choose $T$
- Solve the stationary Malthusian economy with prices $\{w_0, r^K_0, r^L_0, q_0\}$, household functions $\{c_0, x_1, s^K_0, V_0\}$, wealth distribution $\chi_0$, population structure $\mu_0$ and aggregate total wealth $A_0$. Incorporate Malthusian BGP.
- Solve the stationary Solowian economy with prices $\{w_T, r^K_T, r^L_T, q_T\}$, household functions $\{c_T, x_{T+1}, s^K_T, V_T\}$, wealth distribution $\chi_T$, population structure $\mu_T$ and aggregate total wealth $A_T$. Incorporate Solowian BGP.
- Generate the sequence $\{N_t = \sum_{z} \epsilon_z \mu_{z,t}\}_{t=0}^{T}$ iterating forward the initial population structure $\mu_0$ with the population law of motion $\{\Gamma_t\}_{t=0}^{T}$. 

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Computation Algorithm (continued, II)

Given \( \{\hat{K}_t, N_t\}_{t>0}^{T-1} \), generate the sequences of factor prices \( \{w_t, r^K_t, r^L_t\}_{t>0}^{T-1} \).

Given \( \{w_t, r^K_t, r^L_t\}_{t=0}^T \) and \( q_0 \), solve for sequence of the relative price of land \( \{\hat{q}_t\}_{t>0}^{T-1} \) in arbitrage condition (??), a first order difference equation that we solve forward. If \( q_t < 0 \), go back to step 5 and update (raise) the guess \( \{\hat{K}_t\}_{t=0}^T \).

Given \( \{w_t, r^K_t, r^L_t\}_{t=0}^T \), solve for sequence of decision rules \( \{\hat{x}_{t+1}(z,a)\}_{t=0}^{T-1} \) in Euler equation, (7). Since we know the stationary Solowian decision rule, \( x_{T+1}(z,a) \), we can solve for the decision rule at \( T-1 \), \( \hat{x}_T(z,a) \) (a nonlinear function that I approximate using collocation methods) and proceed backwards to obtain the whole sequence \( \{\hat{x}_{t+1}(z,a)\}_{t>0}^{T-1} \).
Given the implied sequence of decision rules \( \{ \hat{x}_{t+1} \}_{t=0}^{T} \), compute the sequence of transitions functions \( \{ \hat{M}_t \}_{t=0}^{T} \). Then, iterate forward the initial distribution \( \chi_0 \) to get the sequence of wealth distributions, \( \{ \hat{\chi}_{t+1} = \hat{M}_t(\hat{\chi}_t) \}_{t=0}^{T} \).

Given the sequence of decision rules, \( \{ \hat{x}_{t+1} \}_{t=0}^{T} \), the sequence of wealth distributions, \( \{ \hat{\chi}_t \}_{t=0}^{T} \), and the sequence of population measures, \( \{ \mu_t \}_{t=0}^{T} \), we can compute the aggregate supply of capital as

\[
\hat{K}_t^S = \hat{A}_{t+1} - \hat{q}_t = \int \hat{a}_{zt,t+1} \mu_{z,t} \, d\hat{\chi}_t - \hat{q}_t
\]
Check the transition of the excess of demand of capital,

$$\max_{1 \leq t < T} |\hat{K}_t - \hat{K}_t^S| < \epsilon$$

(15)

If (15) holds, go to step 12. If not, go back to step 5 and update the guess for the aggregate demand of capital (I use Gauss-Seidel algorithm for updating).

Check if $T$ is large enough. Check if the aggregate supply of capital (chosen at $T - 1$) is close to the aggregate demand of the stationary Solowian economy,

$$|\hat{K}_T^S - K_T| < \epsilon$$

(16)

If (16) holds, we are done. Otherwise go to step 1 and increase $T$. 

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